Economic and Finance statistics

Modelling of yield curves

Block 4 chapter 12

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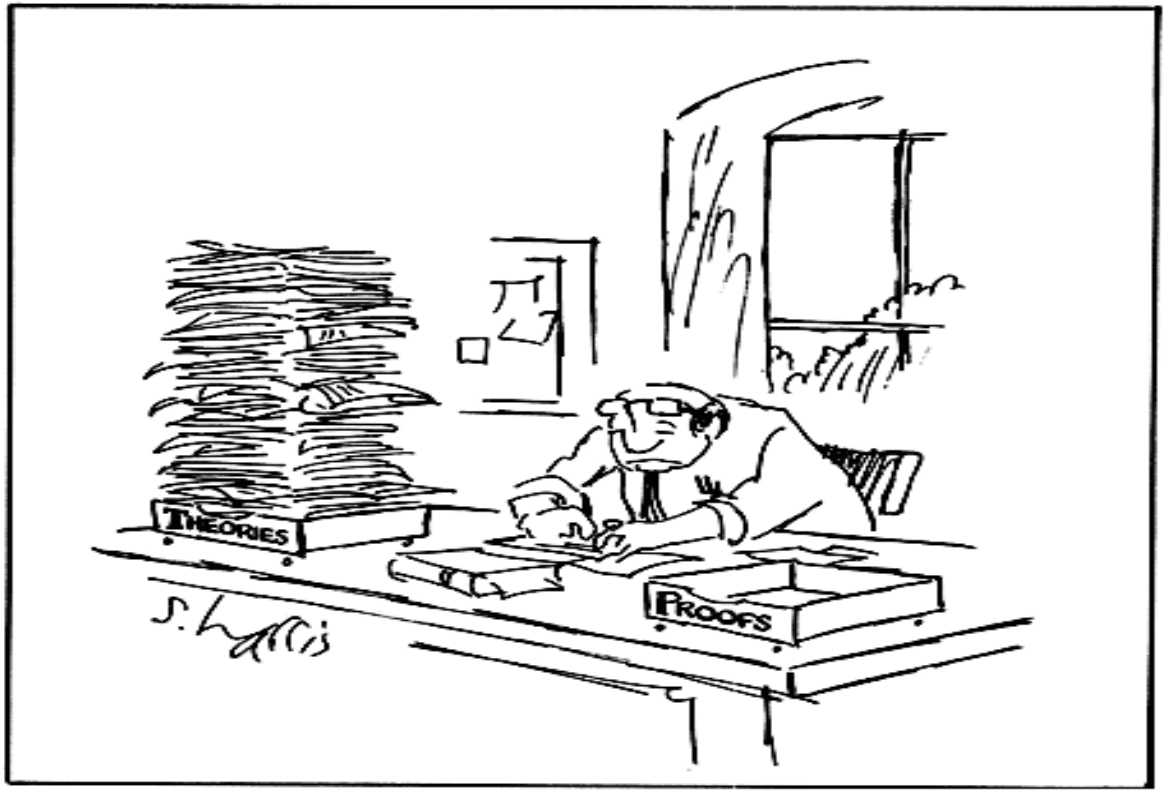
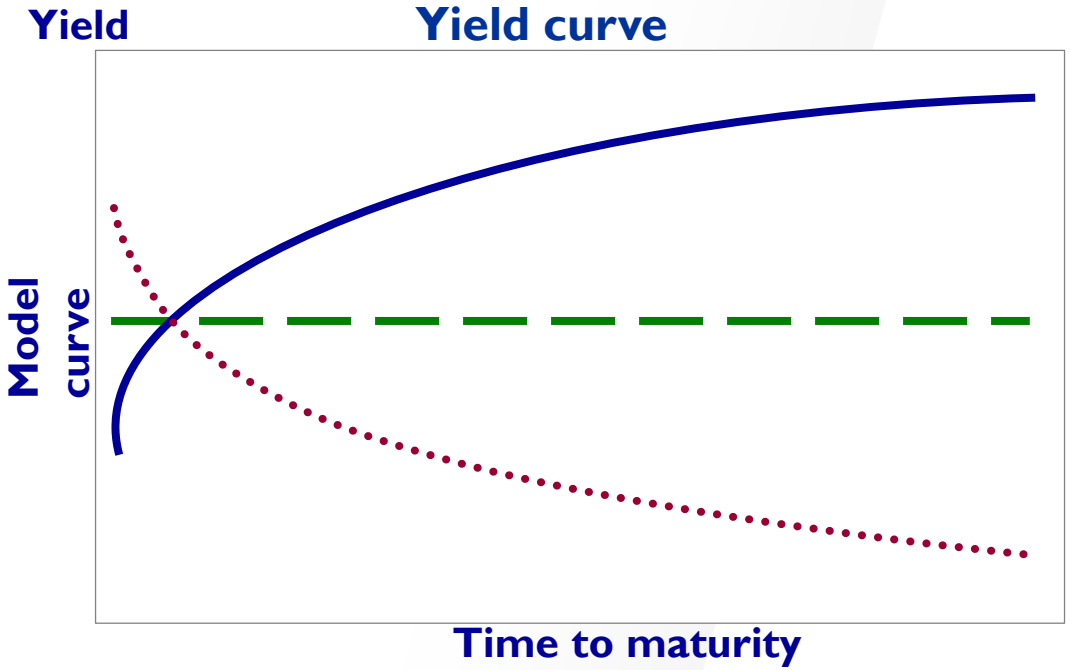
Modelling of yield curves – from theory to practice



Talking statistics: Theory



- Graphical representation of the relationship between yields and the time to maturity
- The slope presents market expectations of short and long-term interest rates
- Provides information on the expected future course of inflation and the pace of real economic activity
- Government securities are free of credit risk and the curve shows the 'price of waiting'

Underlying theories of the term structure of interest rates

Liquidity preference theory – This theory indicates that investors are risk-averse and will demand a premium for holding securities with longer maturities.
Cash today vs deliverable of cash in the future: Demand compensation – risk premia
Therefore all things equal, one would therefore expect to see a rising yield curve.

The pure expectations hypothesis – forward rates govern the curve.
Forward rates are simply expectations of future spot rates and do not take risk premia into account.
Long-term interest rates can act as a predictor of future short-term interest rates.

Instead of buying a long-term bond, an investor could also consider rolling over investments in short term bonds over a period of the same length as the remaining maturity of the long-term bond.

Underlying theories of the term structure of interest rates

The pure expectations hypothesis – (continuation)
Disregarding risk considerations, the total return on the investment in the long-term bond should be equal to the expected cumulative return on the revolving investment in short-term bonds.
This also implies that the average expected future short-term interest rate over the investment horizon should equal the long-term interest rate.

Segmented Markets Hypothesis - The yield curve depends on supply and demand in different sectors and each market segment of the yield curve is only loosely connected to each other

Preferred Habitat – Related to the segmented markets hypothesis, investors may also have a maturity preference, and will shift to another maturity, if the increase in yield is deemed sufficiently compensating the shift.

Models of the term structure of interest rates

Parsimonious parametric models	
Nelson and Siegel (1987)	Exponential approximation to the discount rate function to bond prices
Svensson (1994)	
Spline based models	
McCulloch (1971), (1975)	Cubic spline approximation to the discount rate function with a smoothness penalty
Vasicek and Fong (1982)	Exponential splines to the discount rate function to bond prices
Mastronikola (1991)	Cubic spline approximation to the par yield function
Fischer, Nychka and Zervos (1995)	Cubic spline approximation to the forward rate function with a constant smoothness penalty
Waggoner (1997)	Cubic spline approximation to the forward rate function with a variable smoothness penalty
Anderson and Sleath (2001)	Same as Waggoner with a smoothness penalty which varies over maturities

A trade off between flexibility to accommodate genuine bends in the term structure and "smoothness".

Yield curve models - Parametric models

The N&S model is a parametric model which specifies a functional form for the instantaneous forward rate, f(t) :

$$f(t) = \beta_0 + \beta_1 \exp\left(-\frac{t}{\tau}\right) + \beta_2 \frac{t}{\tau} \exp\left(-\frac{t}{\tau}\right)$$

This model has the ability to capture the stylised facts, describing the behaviour of the forward rate curve.

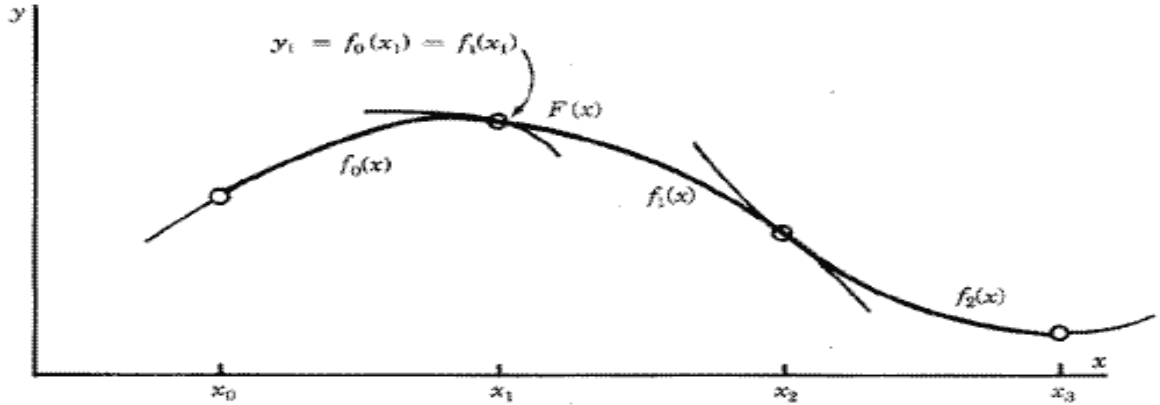
The parameters of these models can be interpreted as follows;

- **β0 is the long term asymptote**
- **β1 is the spread between the long and short term and hence**
- **β0 + β1 is equal to the short term rate (the rate at zero maturity)**
- **Furthermore, τ specifies the position of the first hump or U-shape**
- **β_2 determines the magnitude and direction of the hump**

Yield curve models – Spline based models

Spline- based methods model a curve by a piecewise cubic polynomial, with segments joint at so-called knot points

A spline is a piecewise polynomial function. It possesses a sufficiently high degree of smoothness at the places where the polynomial pieces connect (which are known as knots)



1st functional form for interval X_0 to X_1; 2nd and 3rd for X_1-X_2; X_2-X_3)
Intersections are knot points (circles)

1 Which methods fits best for central banks – ECB?

Parsimonious parametric models	
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2 Which statistics tests and criteria should be applied ?

Flexibility and goodness of fit test

- Statistics test to test if the model estimations adequately captures the movements in the underlying term structure, in particular instruments available for shorter maturities
- Out of sample test (weighted mean average error, & hit rate)

Robustness test

- Statistics test to ensure that changes in the data at one maturity do not have a disproportionate effect on the fit for other maturities
- price changes in long term impact on short term

Smoothness test

- Statistics test ensuring best overall fit without trying to fit every data point
- Comparing spreads between different maturities

Knowing your source and data before starting any testing

12 golden rules for obtaining a quality sample

1. Who is your supplier
2. Type of instruments available
3. Representativeness of bonds per sector
4. Credit risks considerations
5. Liquidity considerations
6. Price types
7. Maturity spectrum
8. Removal of outliers
9. Special effects and adjustments
10. In-sample and out-off sample
11. Sample size
12. Time periods

4 Creating your data sample – results of data testing

source

- Daily bond and price information from mtsmarkets (euromts Ltd)
- Ratings of bonds are provided by Fitch Rating

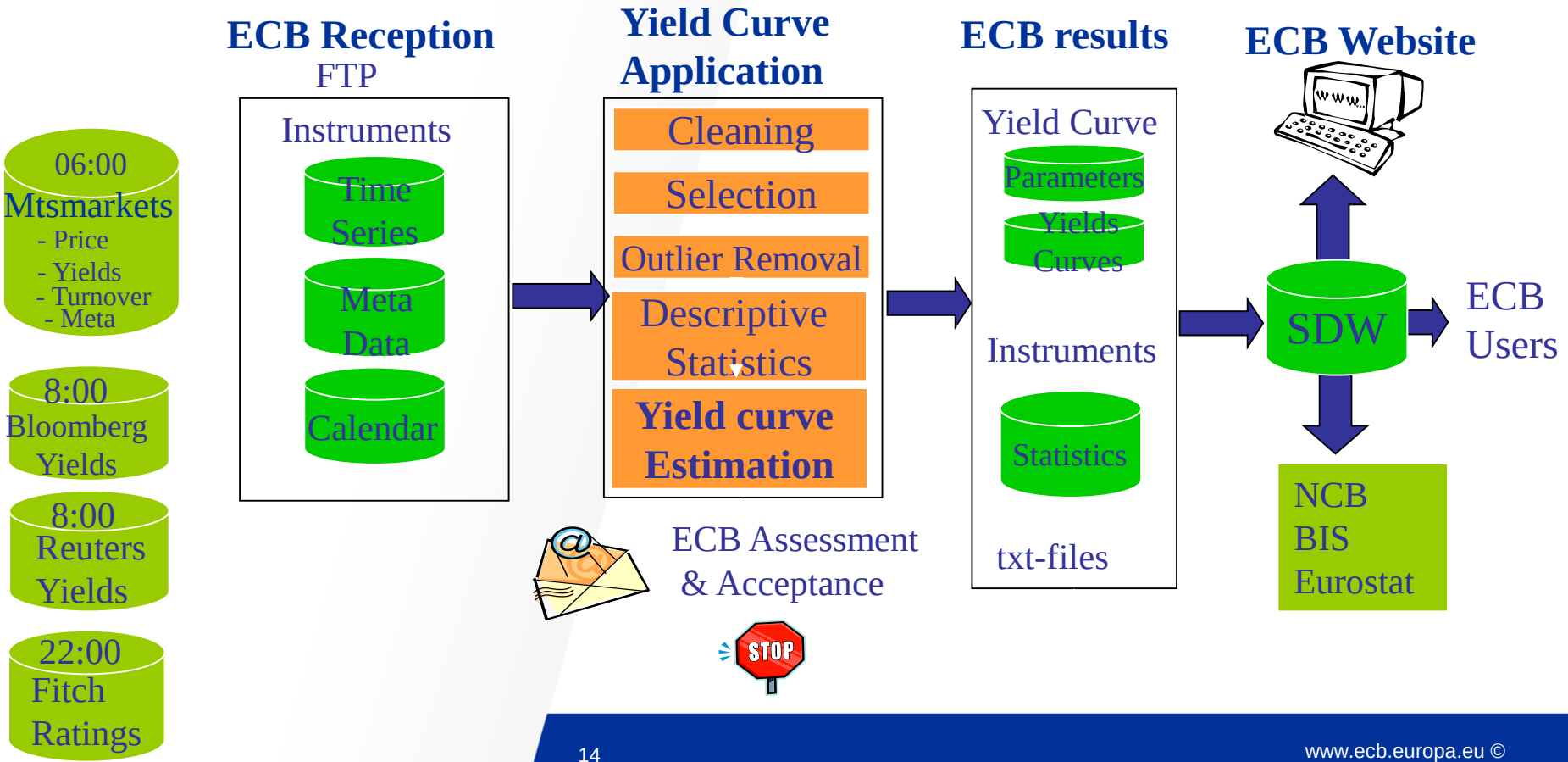
criteria

Only bonds, which

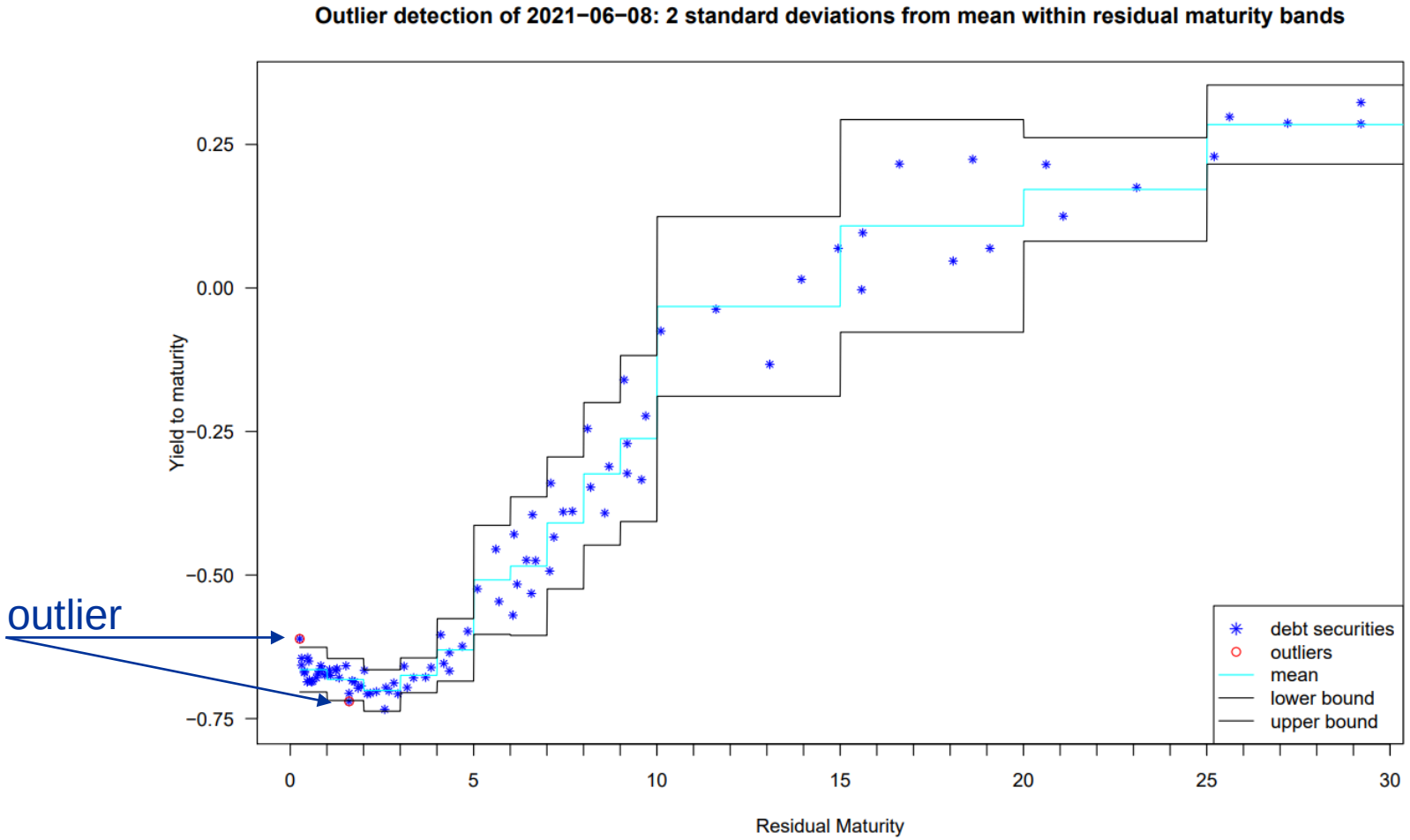
- are issued by central government
- have an outstanding of min EUR 5 Billion
- have no special features (Excl. Brady bonds, convertible bonds)
- have fixed or zero coupon bonds
- are actively traded
- max 3 basis point spread between bid and ask
- residual maturity between 3 months and 30 years

are included within the sample for estimating euro area yields

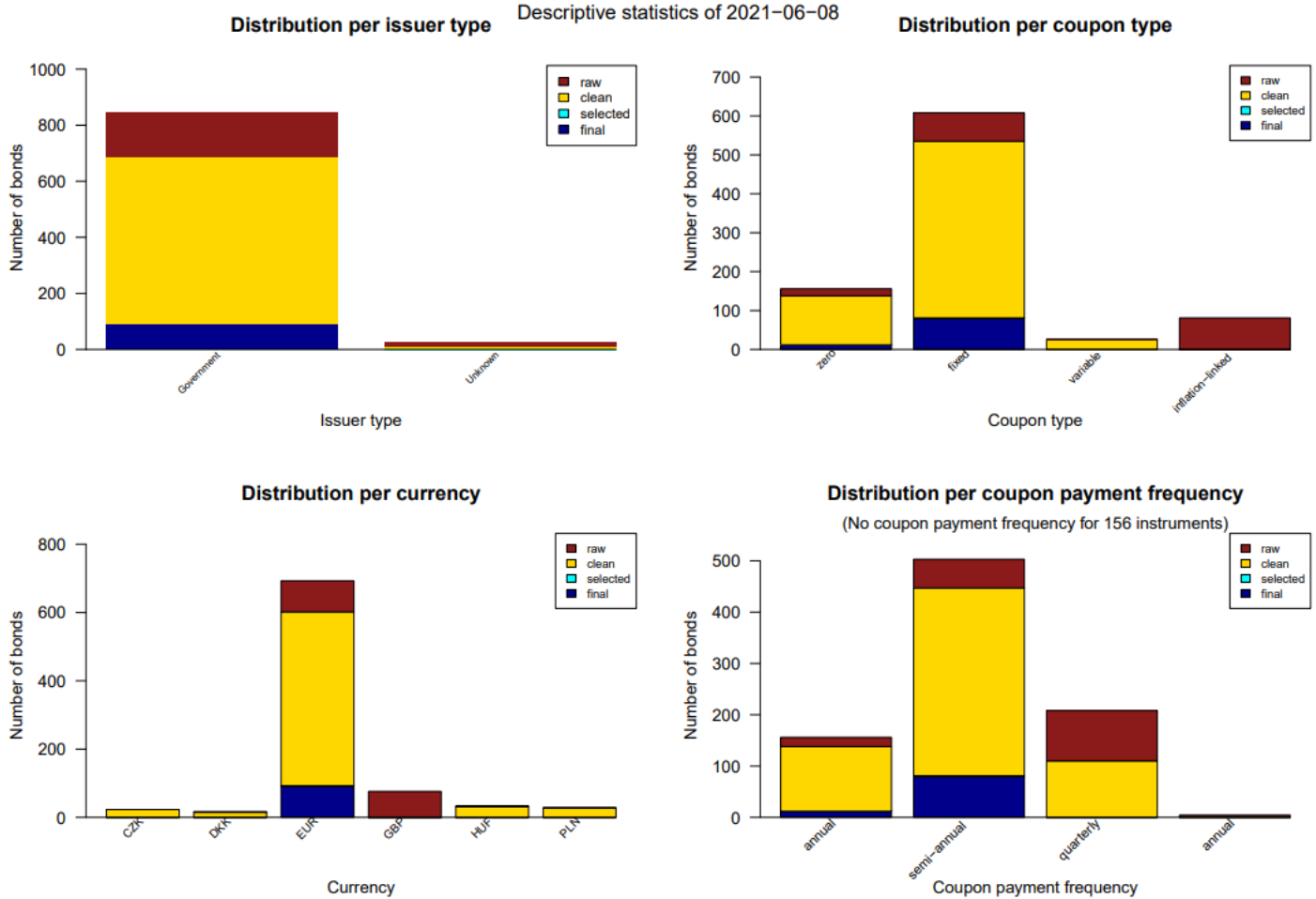
5 Creating your infrastructure for testing and production



Outlier detection: 2 x Standard Deviation - AAA (91 bonds)



Descriptive statistics – from raw data (red) to final dataset (blue)



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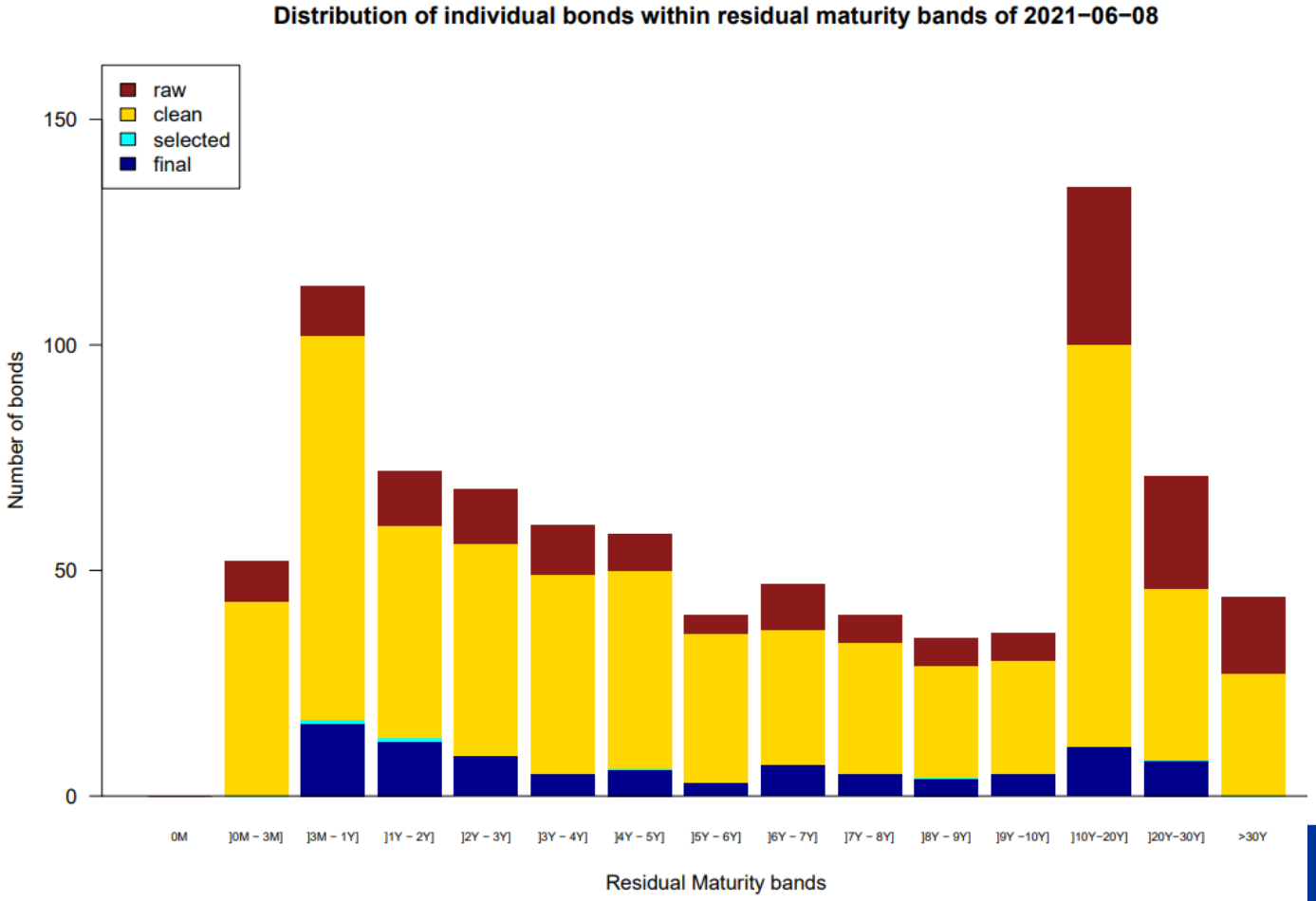

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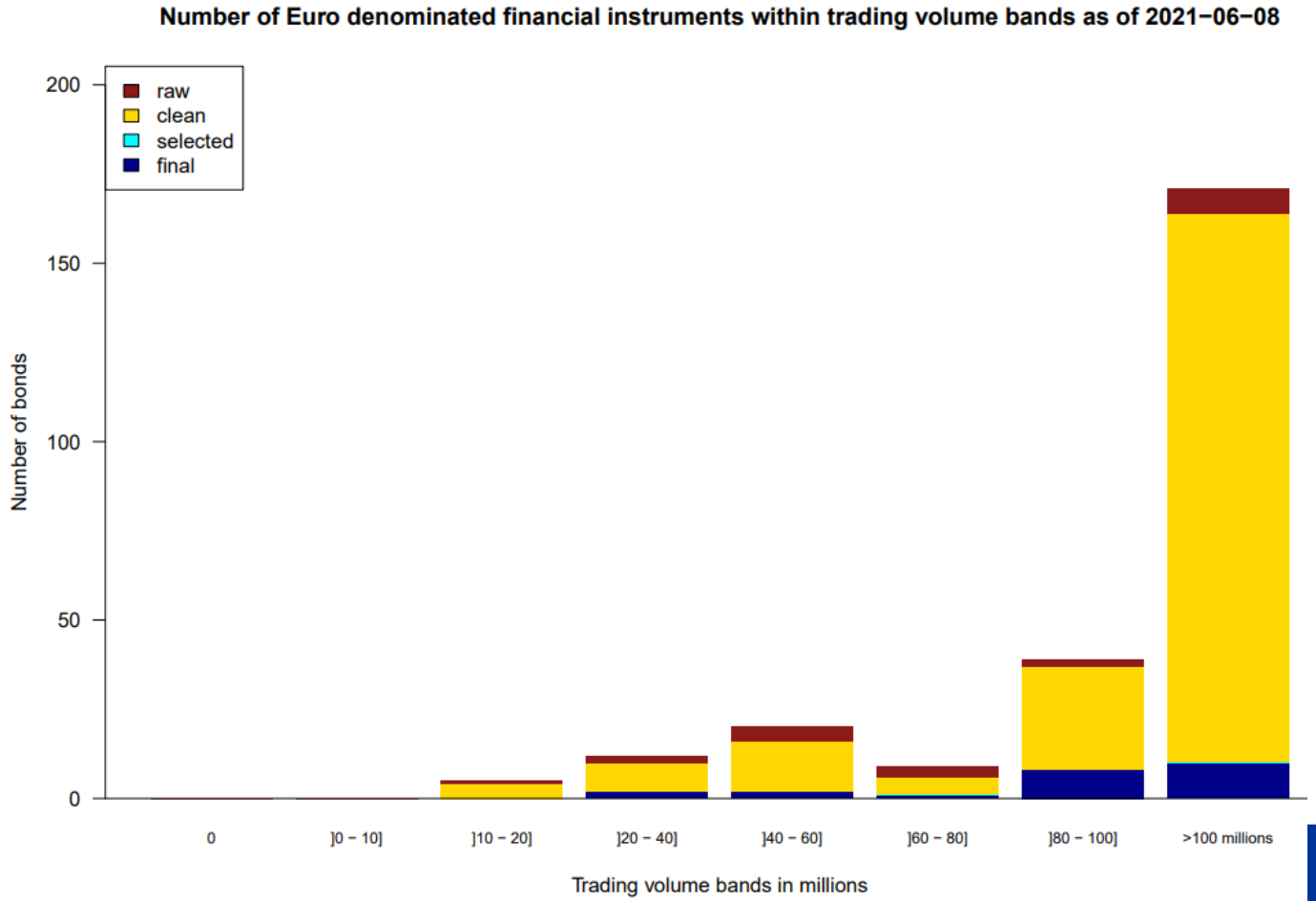
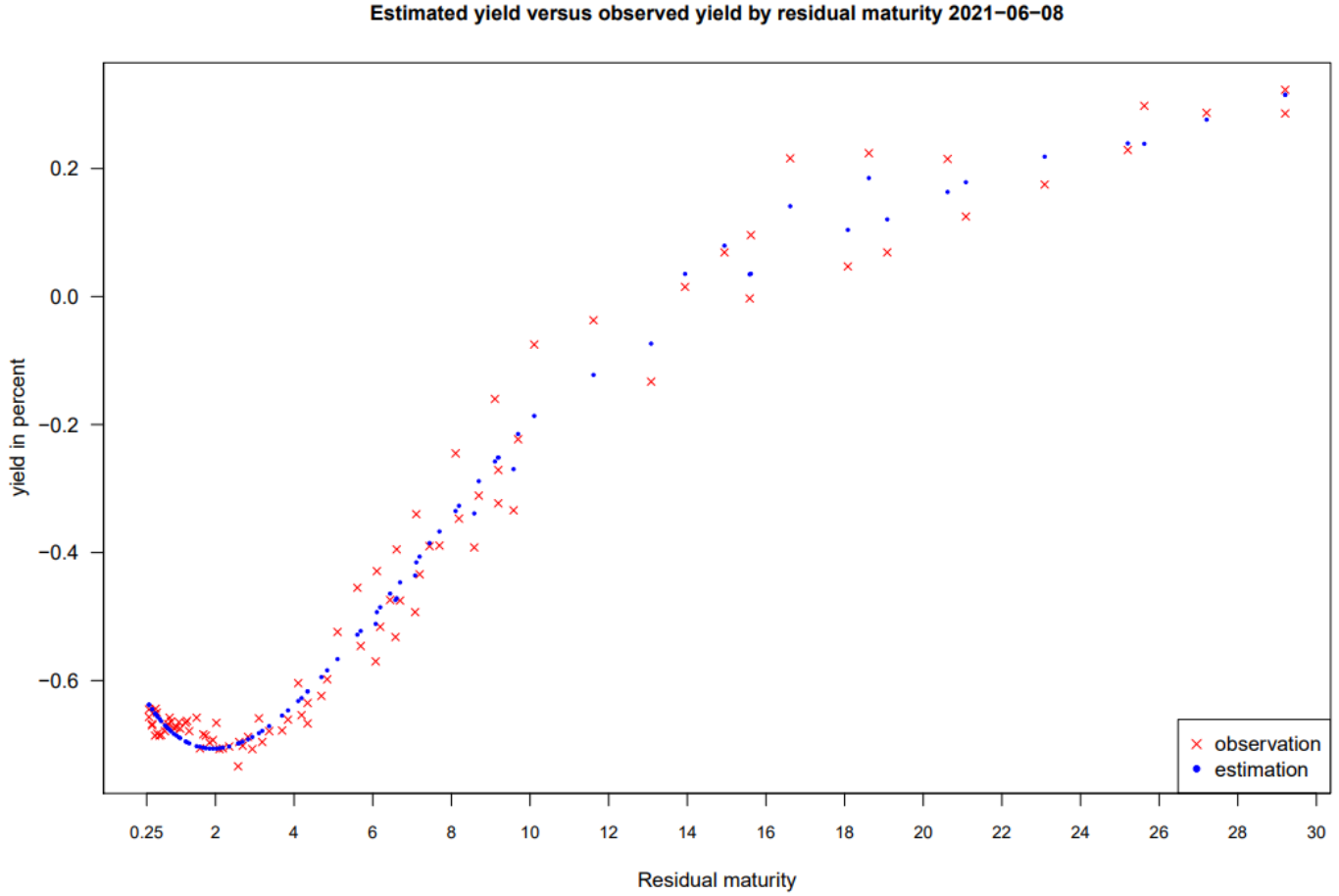


Estimated yield versus observed yield by residual maturity

Date: 2021−06−08 Beta0: 0.22 Beta1: −0.83 Beta2: 13.68 Beta3: −13.87 Tau1: 5.61 Tau2: 4.91



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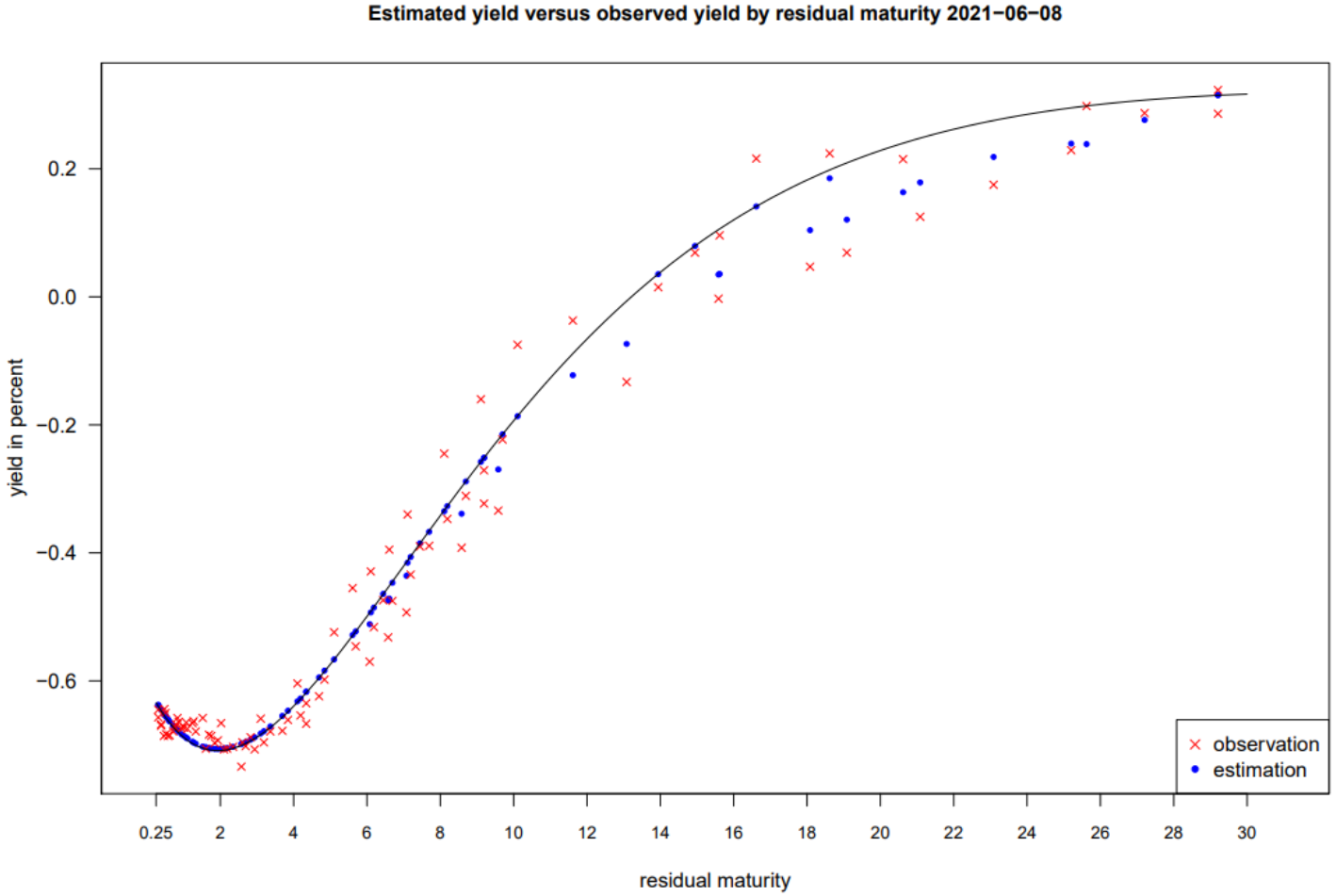


If the links to the movies do not work – use the hyperlink here: Results of model testing

Svensson model movie.avi

Svensson and N&S switching model.avi

Waggoner (VRP) with continuous penalty model

ECB yield curves are released daily at noon at the ECB website:

1. based on market prices and yields of euro area central government bonds denominated in euro
2. covering a remaining time-to-maturity ranging from 3 months to 30 years
3. curves and spreads are based on a selection of the most liquid AAA-rated euro area central government bonds, AA-to-AAA-rated euro area central government bonds and all euro area central government bonds

Additional reading:

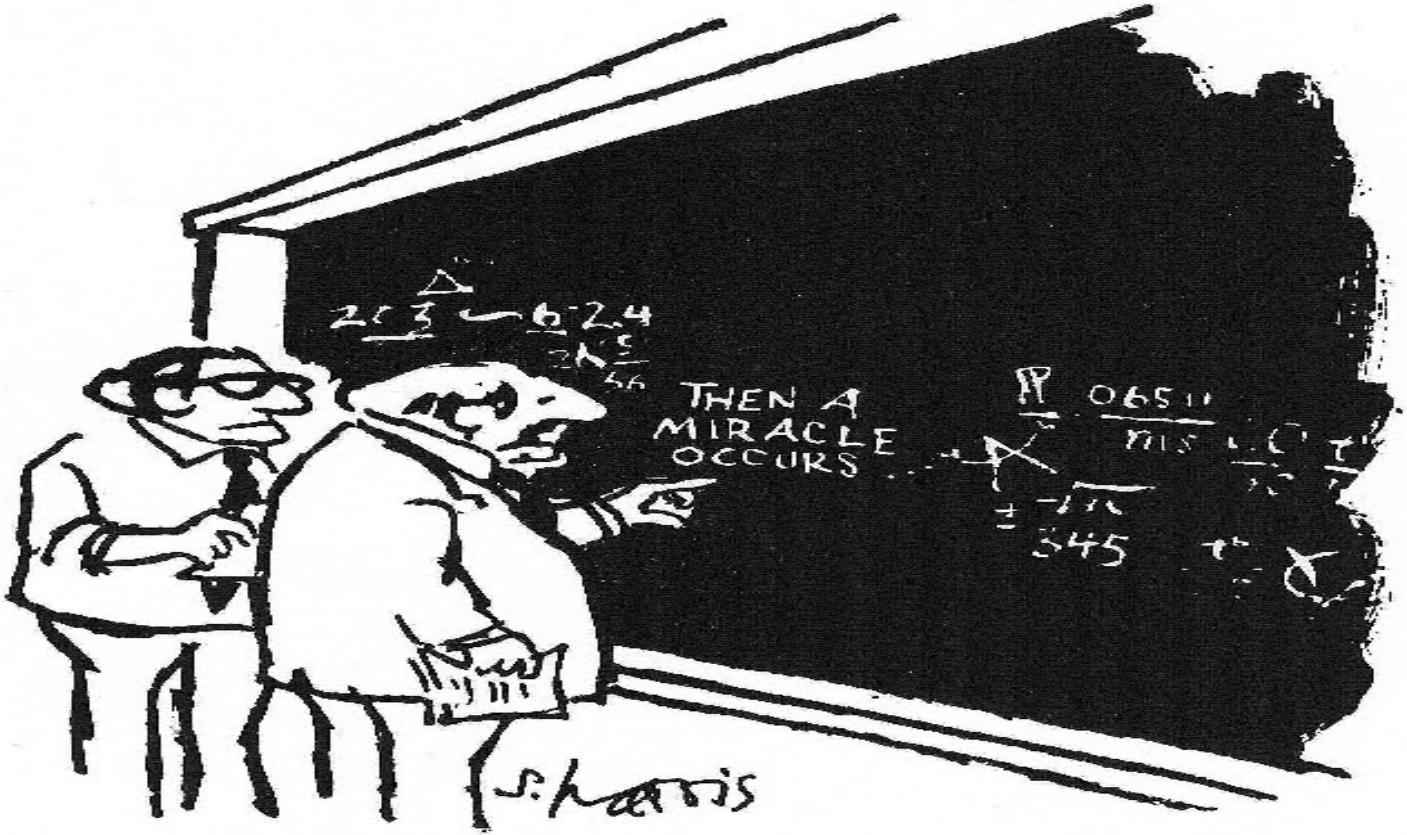
ECB Statistics Paper Series: Yield curve modelling and a conceptual framework for estimating yield curves: evidence from the European Central Bank's yield curves

ECB Monthly Bulletin article: http://www.ecb.europa.eu/pub/pdf/other/pp95-103_mb200802en.pdf

Conclusions

Three takeaways – support and collaborations

- Central Banks & Statistics Offices are part of the official statistical system
- Independent quality statistics requires lead time preparation and supports evidence based decisions
- European collaborative spirit and knowledge centre in statistics, economics & Finance



"I think you should be more explicit here in step two."

Questions and (hopefully) answers

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Fact-check your talk before you walk

WHAT ABOUT YOU WRITING?